

NN, Backpropagation  
Convolutional NN  
Recurrent NN.

## Lecture 23: Recurrent Neural Network (RNN)

[SCS4049-02] Machine Learning and Data Science

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{ 기말고사, 중간고사 시험문제  
시험범위 : + 중간고사 이후 강의  
+ Python도 포함  
(컴퓨터X, 손으로 푸는거)  
비대면으로, → 정리해서 공부.

# Tentative schedule

week	topic	date (수 / 월)
1	Machine Learning Introduction & Basic Mathematics	09.02 / 09.07
2	Python Practice I & Regression	09.09 / 09.14
3	AI Department Seminar I & Clustering I	09.16 / 09.21
4	Clustering II & Classification I	09.23 / 09.28
5	Classification II	(추석) / 10.05
6	Python Practice II & Support Vector Machine I	10.07 / 10.12
7	Support Vector Machine II & Decision Tree and Ensemble Learning	10.14 / 10.19
8	Mid-Term Practice & <b>Mid-Term Exam</b>	10.21 / <b>10.26</b>
9	휴강 & Dimensional Reduction I	10.28 / 11.02
10	Dimensional Reduction II & Neural networks and Back Propagation I	11.04 / 11.09
11	Neural networks and Back Propagation II & III	11.11 / 11.16
12	AI Department Seminar II & Convolutional Neural Network	11.18 / 11.23
13	Python Practice III & Recurrent Neural network ←	11.25 / 11.30
14	Model Optimization & Autoencoders	12.02 / 12.07
15	<b>Final exam</b>	(휴강) / <b>12.14</b>

「길의응답」?!

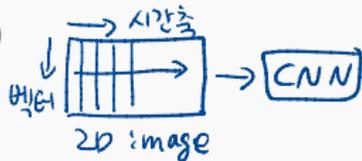
# Introduction of recurrent neural network (RNN)

## Recurrent

- Rumelhart, et. al., Learning Internal Representations by Error Propagation (1986)
- $\{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(t-1)}, \mathbf{x}^{(t)}, \mathbf{x}^{(t+1)}, \dots, \mathbf{x}^{(N)}\}$  와 같은 sequence를 처리하는데 특화 *순서를 갖는*
- RNN은 긴 시퀀스를 처리할 수 있으며, 길이가 변동적인 시퀀스도 처리 가능 *집합상 데이터.*
- Parameter sharing: 전체 sequence에서 파라미터를 공유함

⊗ CNN은 시퀀스를 다룰 수 없나? *가능하지만 트러닝하는 이점.*

- 시간 축으로 1-D convolution  
⇒ 시간 지연 신경망도 가능하지만 깊이가 얕음 (shallow)
- RNN은 깊은 구조(deep)가 가능함



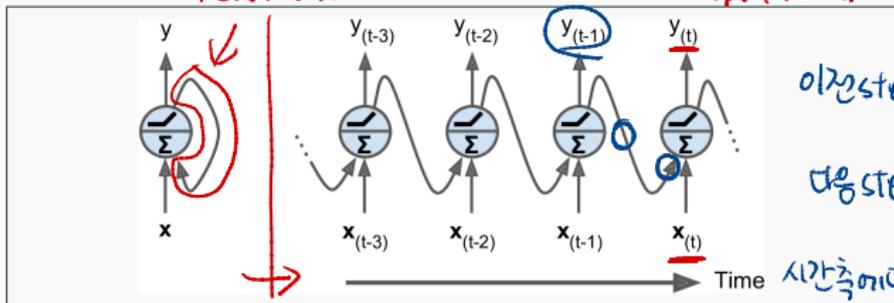
- Data type (RNN) *sequence*
  - 일반적으로  $\mathbf{x}_t$ 는 시계열 데이터 (time series, temporal data)
  - 여기서  $t = 1, 2, \dots, N$ 은 time step 또는 sequence 내에서의 순서
  - 전체 길이  $N$ 은 변동적 *time stamp*

# Recurrent neuron



recurrent!

input/output → 순서 붙여.



이전 step의 output  
↓  
다음 step의 input 줘야.

시간 축에 대해서만 풀어야.

☆ unit 하나씩 ↑  
layer 하나씩 ↓

Figure 14-1. A recurrent neuron (left), unrolled through time (right)

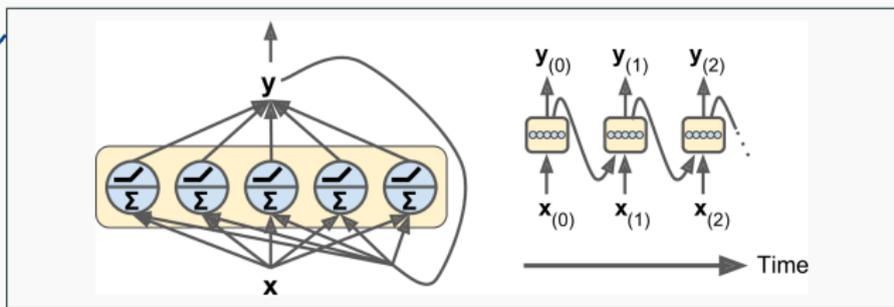


Figure 14-2. A layer of recurrent neurons (left), unrolled through time (right)

- **Unfold or unroll**: 순환적(recurrent, recursive) 계산을 순환이 없는, 그러나 반복적인 구조를 갖는 형태의 그래프로 변환하는 과정

# Recurrent neuron

아래와 같은 그림으로 보여줄 것 → 식으로 표현하면

Output of a single recurrent neuron for a single instance

$$\underline{\mathbf{y}}^{(t)} = \phi \left( \underbrace{\mathbf{x}^{(t)T} \cdot \mathbf{w}_x}_{\text{input} \uparrow} + \underbrace{\mathbf{y}^{(t-1)T} \cdot \mathbf{w}_y}_{\text{앞의 output} \uparrow} + b \right) \quad \text{벡터}^T \text{ 벡터} \rightarrow \text{scalar.} \quad (1)$$

Outputs of a layer of recurrent neurons for all instances in a minibatch

$$\mathbf{Y}^{(t)} = \phi \left( \underbrace{\mathbf{X}^{(t)} \cdot \mathbf{W}_x}_{\text{input}} + \underbrace{\mathbf{Y}^{(t-1)} \cdot \mathbf{W}_y}_{\text{앞의 output}} + \mathbf{b} \right) \quad (2)$$

$$= \phi \left( \left[ \begin{array}{cc} \mathbf{X}^{(t)} & \mathbf{Y}^{(t-1)} \end{array} \right] \cdot \mathbf{W} + \mathbf{b} \right) \quad \text{with } \mathbf{W} = \begin{bmatrix} \mathbf{W}_x \\ \mathbf{W}_y \end{bmatrix} \quad (3)$$

[input    앞의 output]  
? 같이 하나씩 하나씩

$$\begin{bmatrix} \mathbf{x}^{(t)} & \mathbf{y}^{(t-1)} \end{bmatrix} \begin{bmatrix} \mathbf{W}_x \\ \mathbf{W}_y \end{bmatrix} \\ = \mathbf{x}^{(t)} \cdot \mathbf{W}_x + \mathbf{y}^{(t-1)} \cdot \mathbf{W}_y$$

# Memory cells and input/output sequences

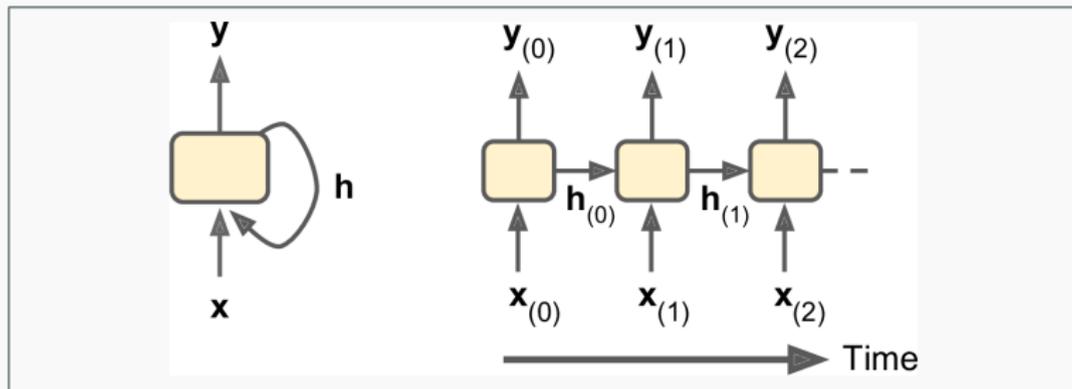


Figure 14-3. A cell's hidden state and its output may be different

# Memory cells and input/output sequences

input/output 이랑 서로 다른 길이가 가능함.

단어의 길이가 같을 수도 있음.

vector  
↓  
seq.

seq.  
↓  
vector

서로 다른 길이의 sequence

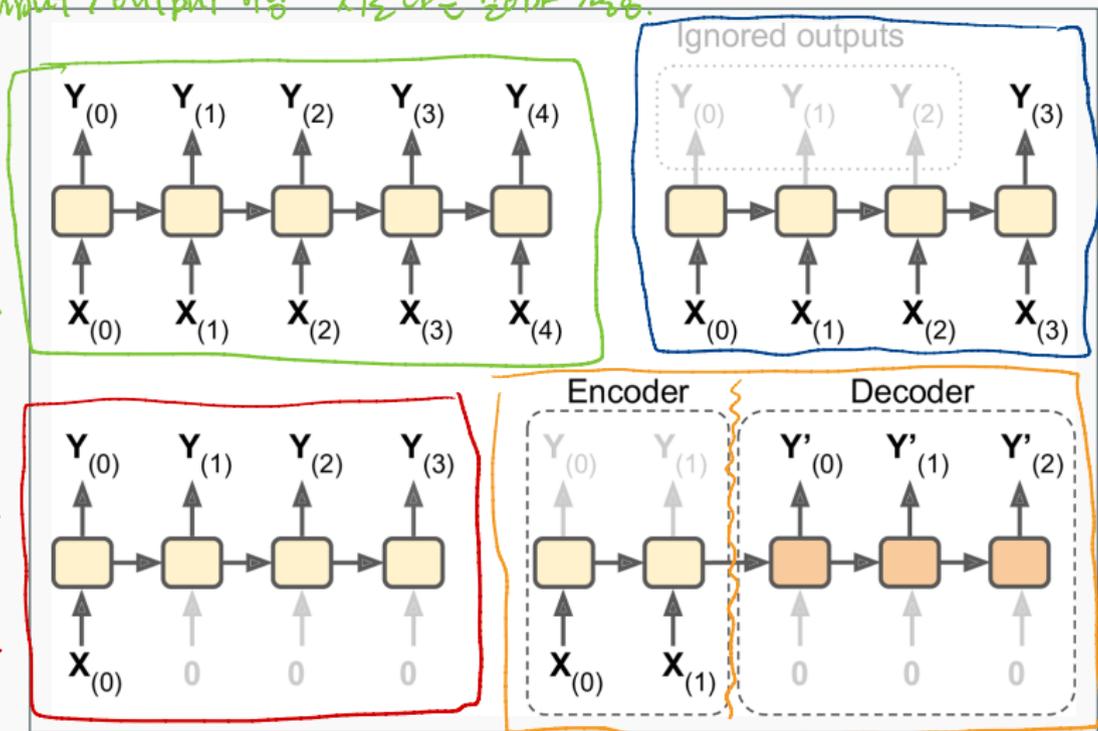


Figure 14-4. Seq to seq (top left), seq to vector (top right), vector to seq (bottom left), delayed seq to seq (bottom right)

# An encoder-decoder network for machine translation

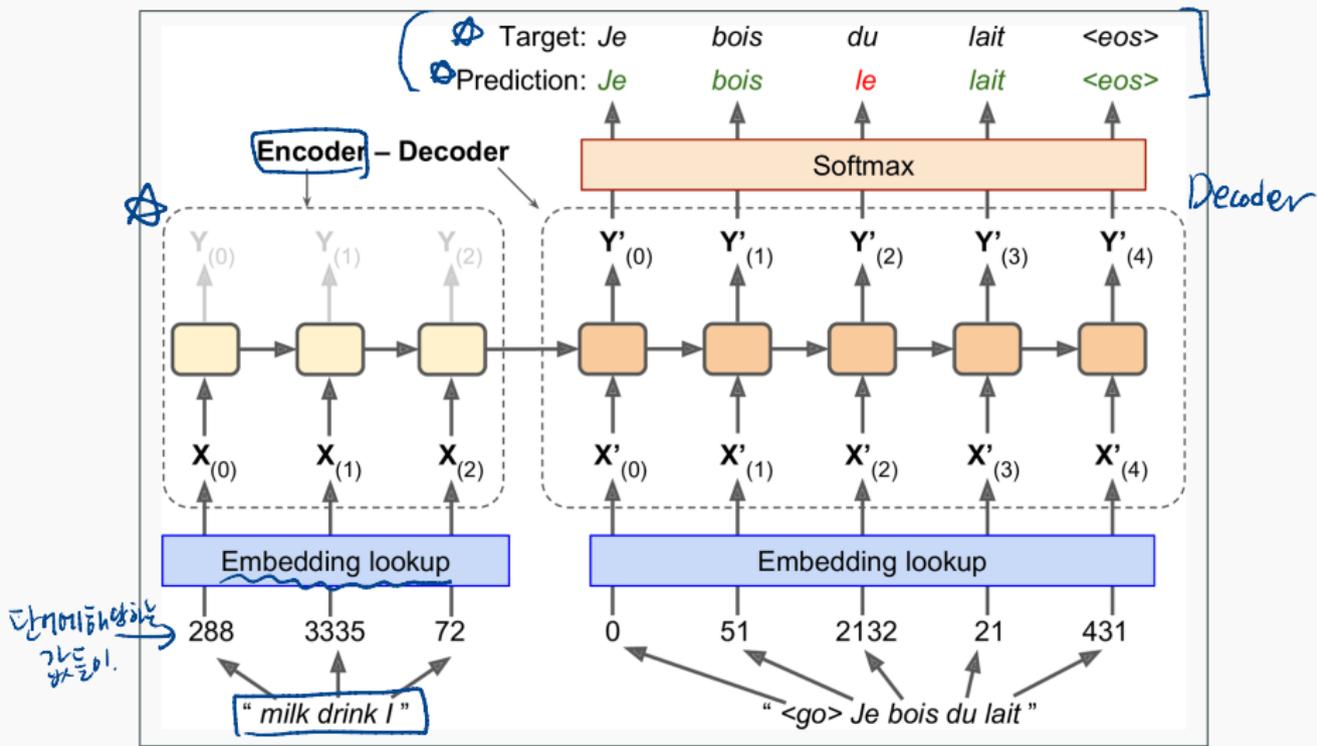


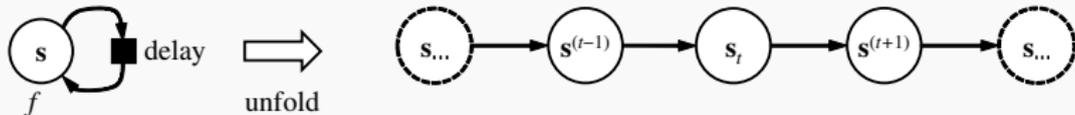
Figure 14-15. A simple machine translation model

## Recurrent neuron

unfolding

$$\underline{s^{(t)}} = \underline{f(s^{(t-1)}; \theta)} \quad (4)$$

↪ parameter

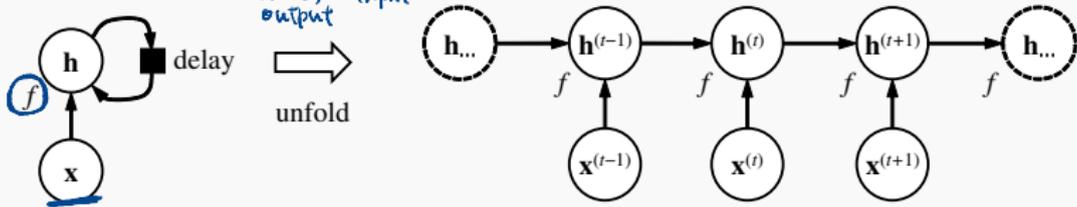


## Recurrent neuron with hidden unit

$$\underline{h^{(t)}} = \underline{f(h^{(t-1)}, x^{(t)}; \theta)} = \underline{g^{(t)}(x^{(t)}, x^{(t-1)}, \dots, x^{(2)}, x^{(1)})} \quad (5)$$

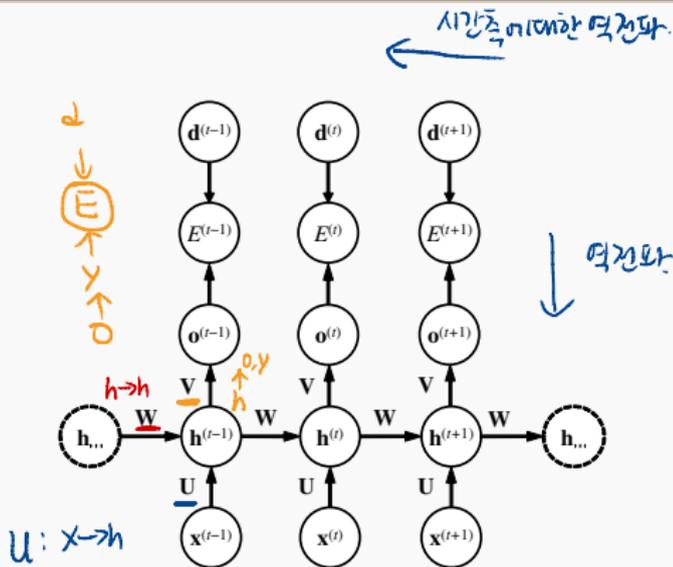
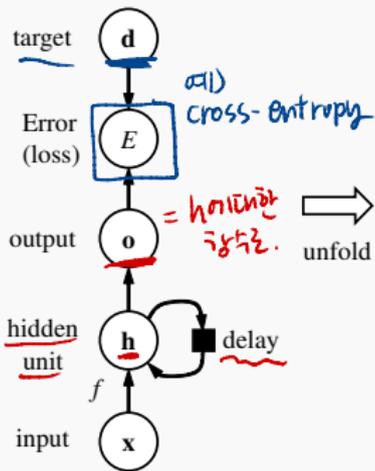
↪ 모든 과거에 동일하게 함수.      모든 과거의 입력에 대한 함수.

↪ 이전 hidden output      ↪ 지금의 input



# Graph representation of RNN

Supervised learning



- 가중치 매트릭스  $U$ ,  $V$ ,  $W$  or  $W_{xh}$ ,  $W_{hy}$ ,  $W_{hh}$
- Error  $E =$  예측값과 참값 사이의 cross entropy with  $y = \text{softmax}(o)$

# Various architectures of RNN



one to one

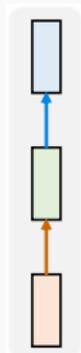


Image Classification  
(Vanilla Neural Networks)

CNN, NN.

one to many

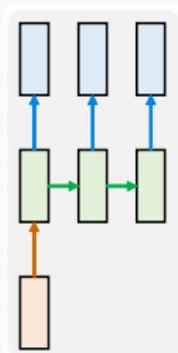
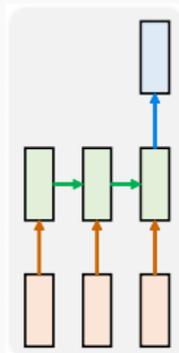


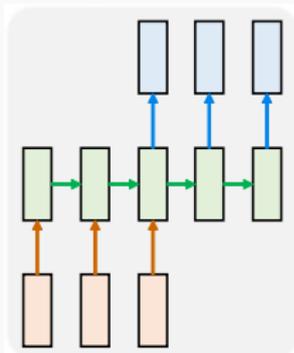
Image Captioning  
(image  $\rightarrow$  seq. of words)

many to one



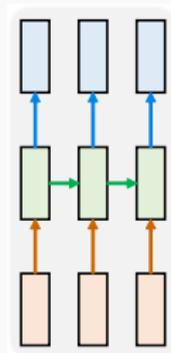
Sentiment Analysis  
(seq. of words  $\rightarrow$  sentiment)

many to many



Machine Translation  
(seq. of words  $\rightarrow$  seq. of words)

many to many



Synced Sequence  
(video classification on frame level)



# Forward propagation

Assuming the initial state  $\mathbf{h}^{(0)} = \mathbf{0}$

weight sum  $\rightarrow \mathbf{a}^{(t)} = \mathbf{W}\mathbf{h}^{(t-1)} + \mathbf{U}\mathbf{x}^{(t)} + \mathbf{b}$  (6)

hidden output  $\rightarrow \mathbf{h}^{(t)} = \tanh(\mathbf{a}^{(t)})$   $\tanh$ : activation. (7)

(visible) output  $\rightarrow \mathbf{o}^{(t)} = \mathbf{V}\mathbf{h}^{(t)} + \mathbf{c}$   $\mathbf{V}$ :  $h \rightarrow y$  (8)

관측 출력  $\rightarrow \mathbf{y}^{(t)} = \text{softmax}(\mathbf{o}^{(t)})$  (9)

Alternatively

$$\mathbf{h}^{(t)} = \tanh(\mathbf{W}_{hh}\mathbf{h}^{(t-1)} + \mathbf{W}_{xh}\mathbf{x}^{(t)})$$
 (10)

$$\mathbf{y}^{(t)} = \text{softmax}(\mathbf{W}_{hy}\mathbf{h}^{(t)})$$
 (11)

Error (loss) function: cross entropy

단일 unit에 대한 손실,  $E^{(t)}(\mathbf{y}^{(t)}, \mathbf{d}^{(t)}) = -\mathbf{d}^{(t)} \log \mathbf{y}^{(t)} = -\sum_{i=1}^D d_i^{(t)} \log y_i^{(t)}$  (12)

$E(\mathbf{y}, \mathbf{d}) = -\sum_{t=1}^N \mathbf{d}^{(t)} \log \mathbf{y}^{(t)}$  (13)

# Deep RNNs

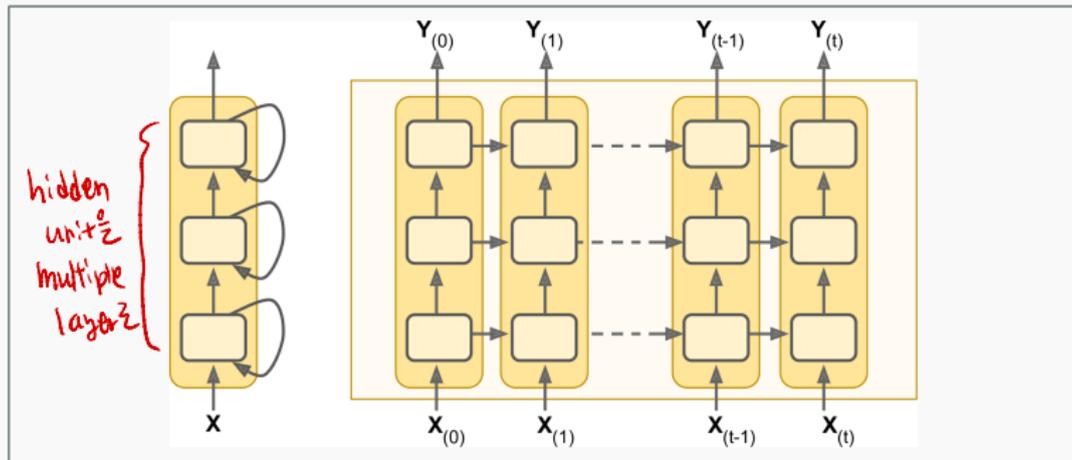
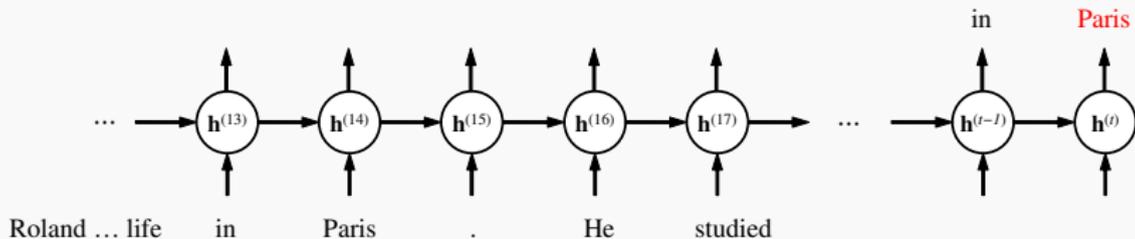


Figure 14-12. Deep RNN (left), unrolled through time (right)

# Long-term dependency problem

⇒ Roland Dyens was born in Tunisia and lived most of his life in Paris.

He studied with Spanish classical guitarist Alberto Ponce and with Désiré Dondeyne. As a performer, Dyens was known for improvisation. Sometimes he opened his concerts with an improvised piece, and he might improvise the program itself, without planning or announcing beforehand what he would be playing. He said that a journalist once told him he had the hands of a classical musician and the head of a jazz musician. He played Bach suites and he played with jazz musicians at the Arvika Festival in Sweden. A heavy metal band did a version of the third movement of his Libra Sonatine. He is still living in where?



질문 단어로 부터 'Paris'는 116 시간 스텝이나 멀리 떨어져 있다.



<sup>6</sup> Gradient는 그렇게까지 멀리 역전파되지 못한다.  
(Vanishing/Exploding Gradient)

seq.의 길이가 길면 → 멀리 떨어진 애들끼리의 영향이 감소

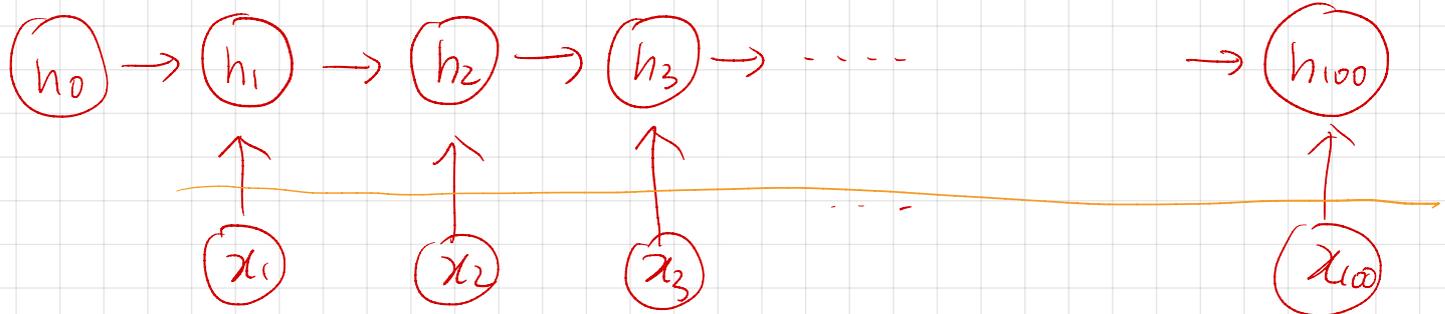
$$a_1 = W_{hh} h_0 \quad a_2 = W_{hh} h_1$$

$$h_1 = \tanh(a_1) \quad h_2 = \tanh(a_2)$$

hidden unit  $\frac{c}{2}$  at.

$$a_{100} = W_{hh} h_{99} = W_{hh} \tanh(a_{99})$$

$$h_{100} = \tanh(a_{100})$$



$$a_{100} = \underbrace{W_{hh}} \tanh \left( \underbrace{W_{hh}} \tanh \left( \underbrace{W_{hh}} \dots \right) \right)$$

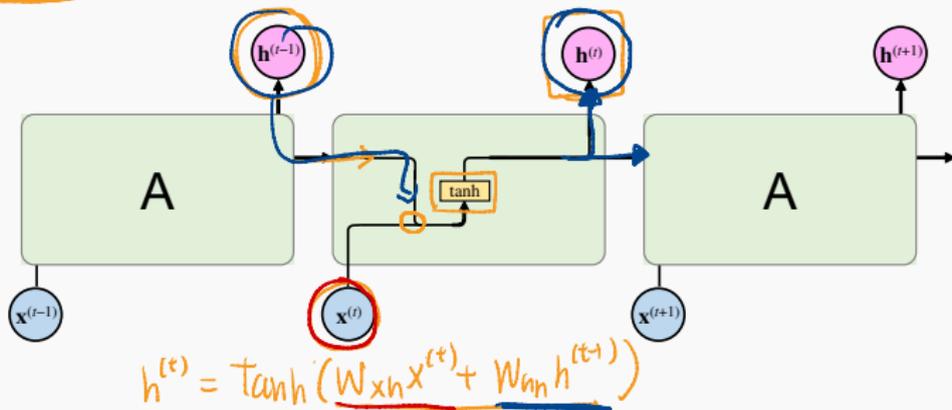
$$\approx \underbrace{W_{hh}}_{\text{2번}}$$

$$a_0 \rightarrow a_{100} \approx \underbrace{W_{hh}}^{100}$$

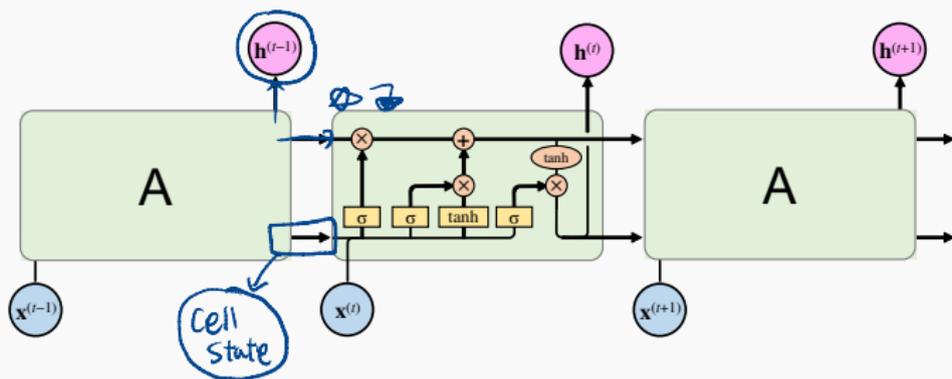
요건관계:  $W_{hh}$  가 지속적으로  
 0이 아니게 되면

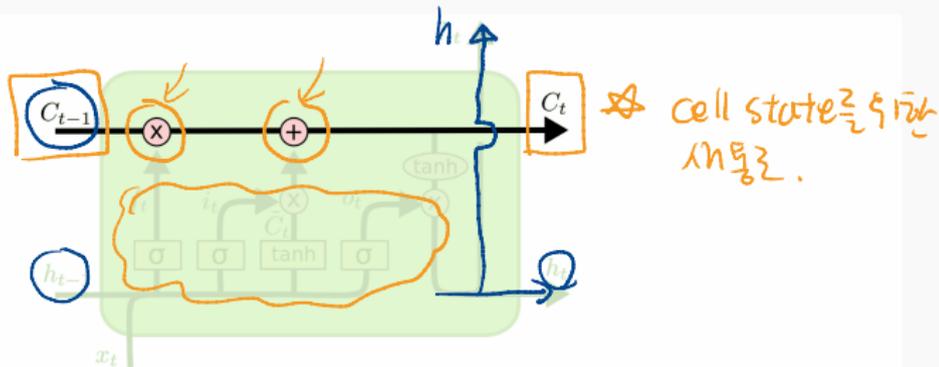
# Long short-term memory (LSTM)

Vanilla RNN

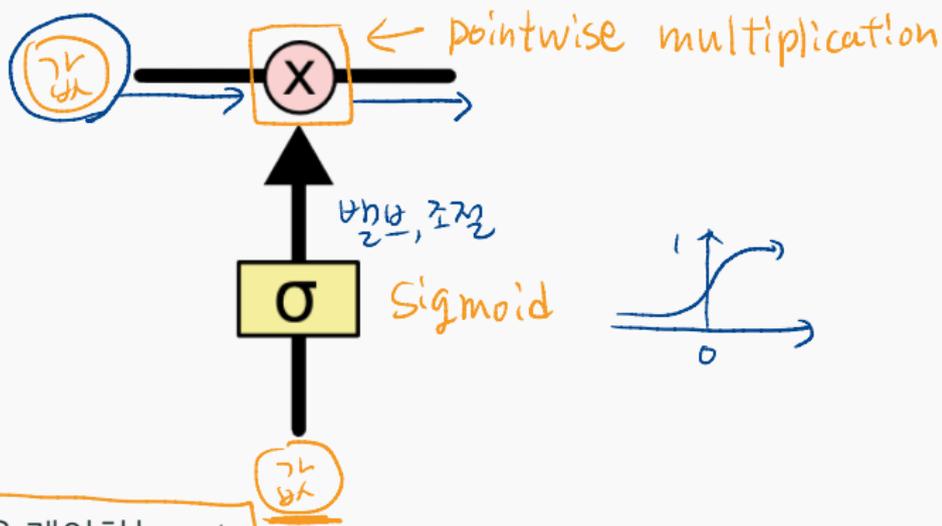


LSTM



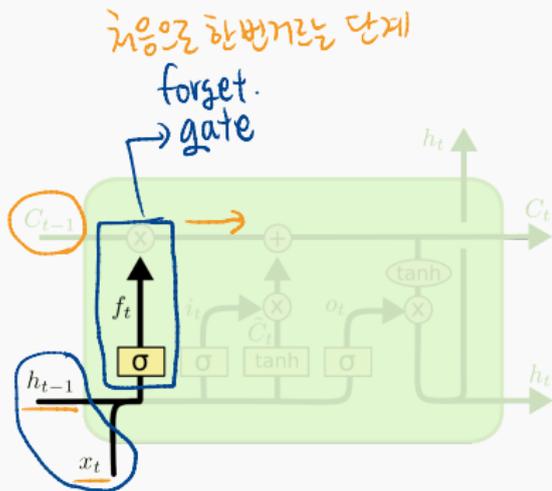


- 컨베이어 벨트 같은 통로가 있어서, 정보가 전체 연결망을 그대로 흘러다닐 수 있음
- 정보가 변경 없이 그대로 흘러다닐 수 있음
- Cell state에 정보를 지우거나 더할 수 있음



- 정보의 흐름을 제어하는 gate
  - Sigmoid 함수  $\sigma$ 와 pointwise multiplication으로 이루어져 있음
  - Sigmoid의 출력에 따라
    - $\sigma = 0$ 일 때, 흐름을 중단함
    - $\sigma = 1$ 일 때, 흐름을 모두 통과
- 반반은 정도를 조절.

# Forget gate layer

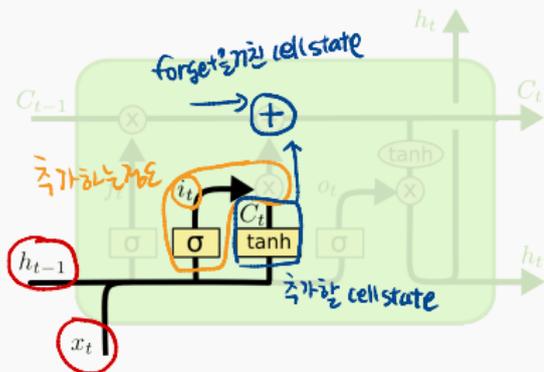


$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

↑                      ↑  
앞의                      지금의  
hidden                      input  
state

- Output of sigmoid  $f_t \in [0, 1]$ 
  - 1: completely keep this
  - 0: completely get rid of this

# Input gate layer



얼마나 추가시킬지 조절

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

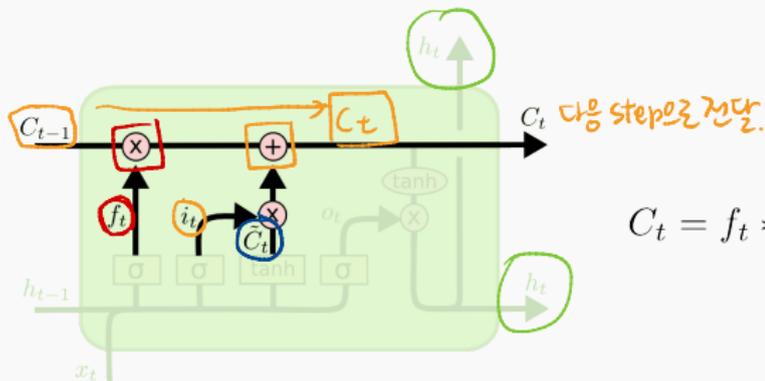
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

조절하는데, 보낼 값

- 어떤 새로운 정보를 입력시킬 지 결정
- 입력 게이트는 어떤 값을 업데이트 할 지 결정
- tanh 층은 새로운 값의 후보가 되는 벡터를 생성

$$i_t \cdot \tilde{C}_t$$

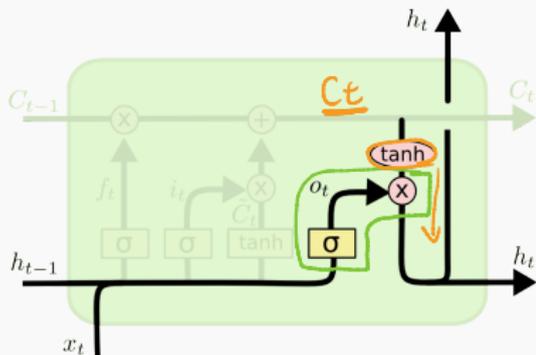
## Update cell state



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

- Cell state가 업데이트 되어 다음 시간 스텝으로 넘어가는 과정  
동시에 새로운 hidden state를 출력하기 위해  $\tanh$ 로 전달됨

# Update ~~cell state~~ hidden state.



$$o_t = \sigma(W_o [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh(C_t)$$

$$h_t \leftarrow C_t, o_t$$

$\rightarrow \tanh \rightarrow \text{gate} \downarrow$

- 현재의 cell state 중에서 출력  $h$ 로 내보낼 값을 선택
- 이 출력은 윗 층과 다음 시간 스텝으로 동시에 값을 보냄

# Appendix

---

## Reference and further reading

- “Chap 14 | Recurrent Neural Networks” of A. Geron, Hands-On Machine Learning with Scikit-Learn and TensorFlow
- “Lecture 11 | Recurrent Neural Networks” of Kwang Il Kim, Machine Learning (2019)
- Christopher Olah, Understanding LSTM Networks ([link](#))