Inclass 05: Mixture of Gaussian Clustering

[SCS4049] Machine Learning and Data Science

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- MoG clustering → 정리.
- meshgrid, MoG (2-D) 에서 확률분포를 표현.
- python.

γ : $K \times N$ 행렬.

k번째 cluster (Gaussian), n번째 샘플.

$$\frac{\pi_k \mathcal{N}(x \mid \mu_k, \Sigma_k)}{\sum_j \pi_j \mathcal{N}(x \mid \mu_j, \Sigma_j)}$$

- $N_k = \sum_{n=1}^{N} \gamma(z_{nk})$, k번째 cluster의 전체 responsibility의 합.

$$\mu_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma(z_{nk}) x_n$$

μ_k : 벡터. $\gamma(z_{nk})$: scalar (0~1). x_n : 벡터

$$\Sigma_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma(z_{nk}) \{(x_n - \mu_k)(x_n - \mu_k)^T\}$$

$\gamma(z_{nk})$: Scalar.

$(x_n - \mu_k)$: 열벡터, $D \times 1$

$(x_n - \mu_k)^T$: 행벡터, $1 \times D$

$\{D \times D$ 행렬$\}$

$\gamma \cdot res.$

res.

$$\pi_k = \frac{N_k}{N}$$

주어진것

계산하는것, update.

E-step.

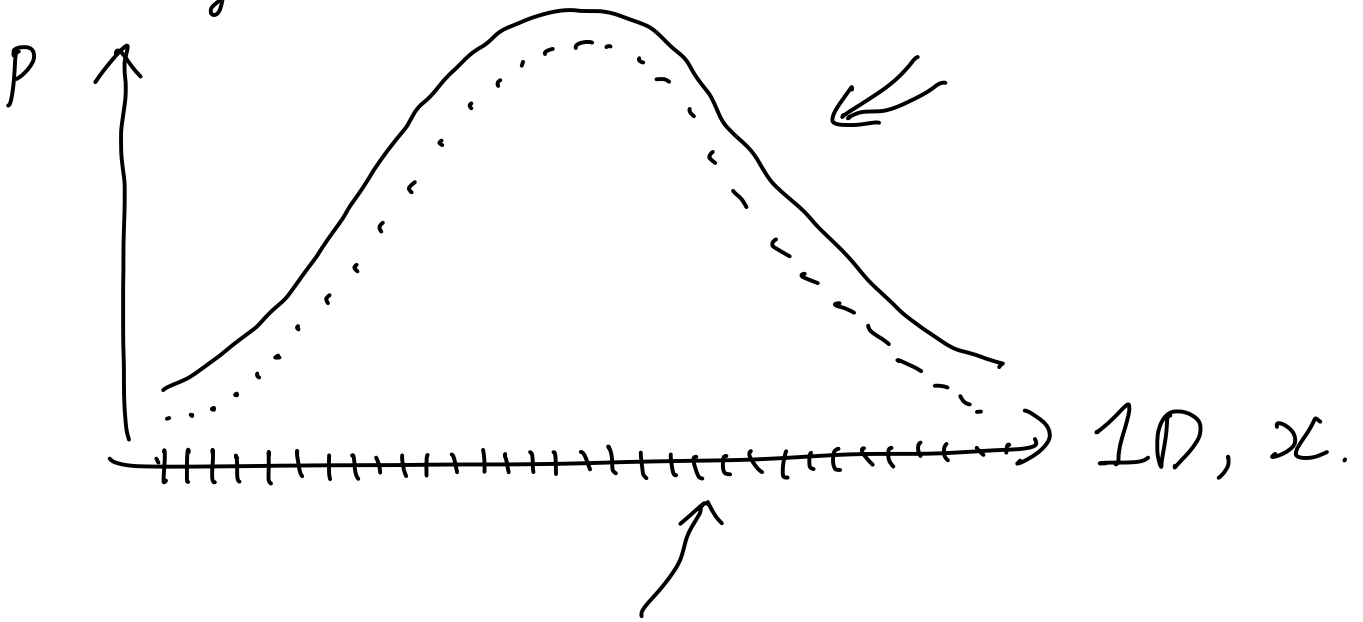
$X, \pi_k, \mu_k, \Sigma_k, \longrightarrow \gamma$

iterative.

M-step.

$X, \gamma \longrightarrow \pi_k, \mu_k, \Sigma_k.$

meshgrid.

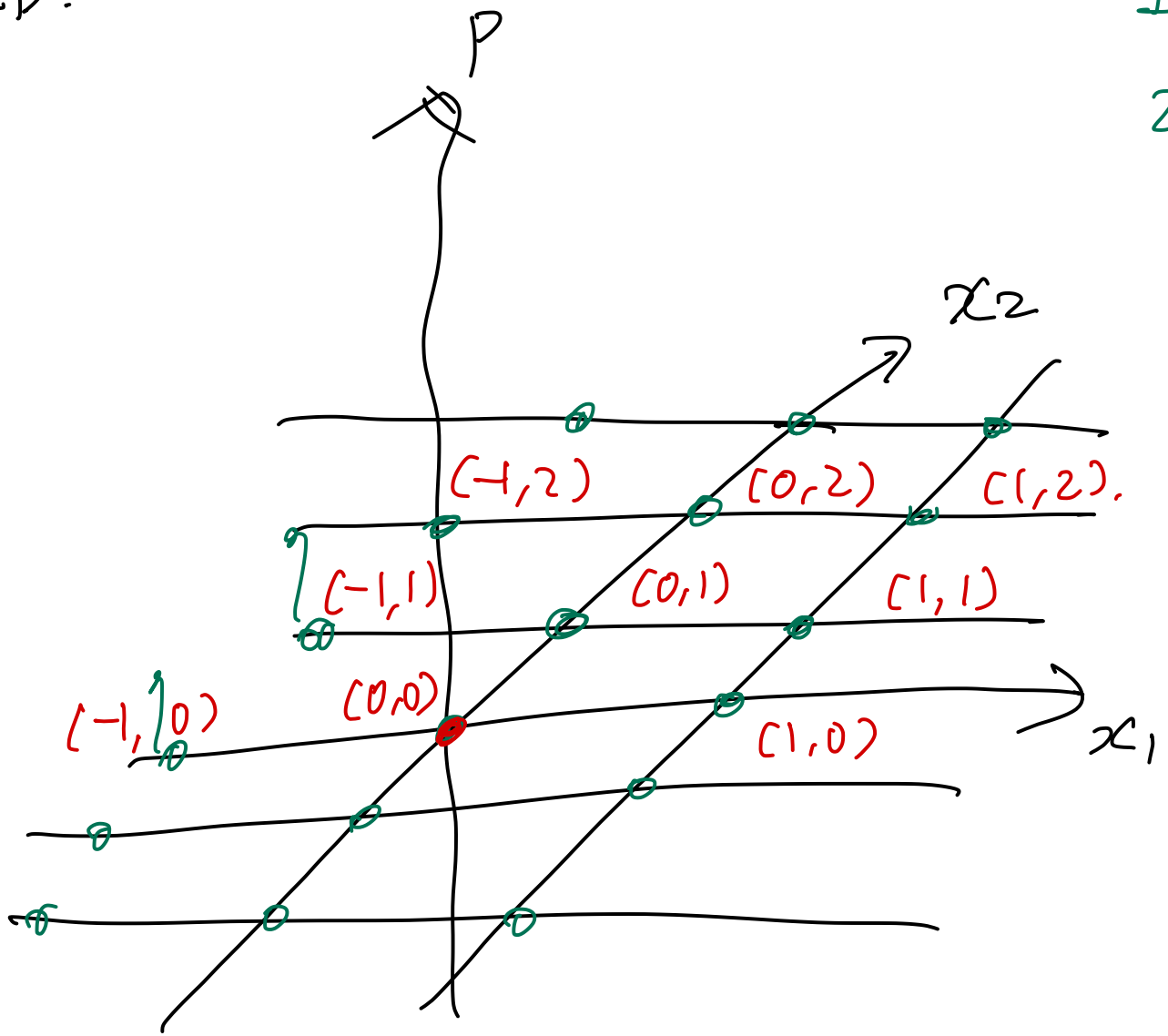


X = np.linspace(0, 1, 100).

$$p = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

plot(x, p)

2D.



1D

2D.

해당될 해당될 벡터. 벡터.

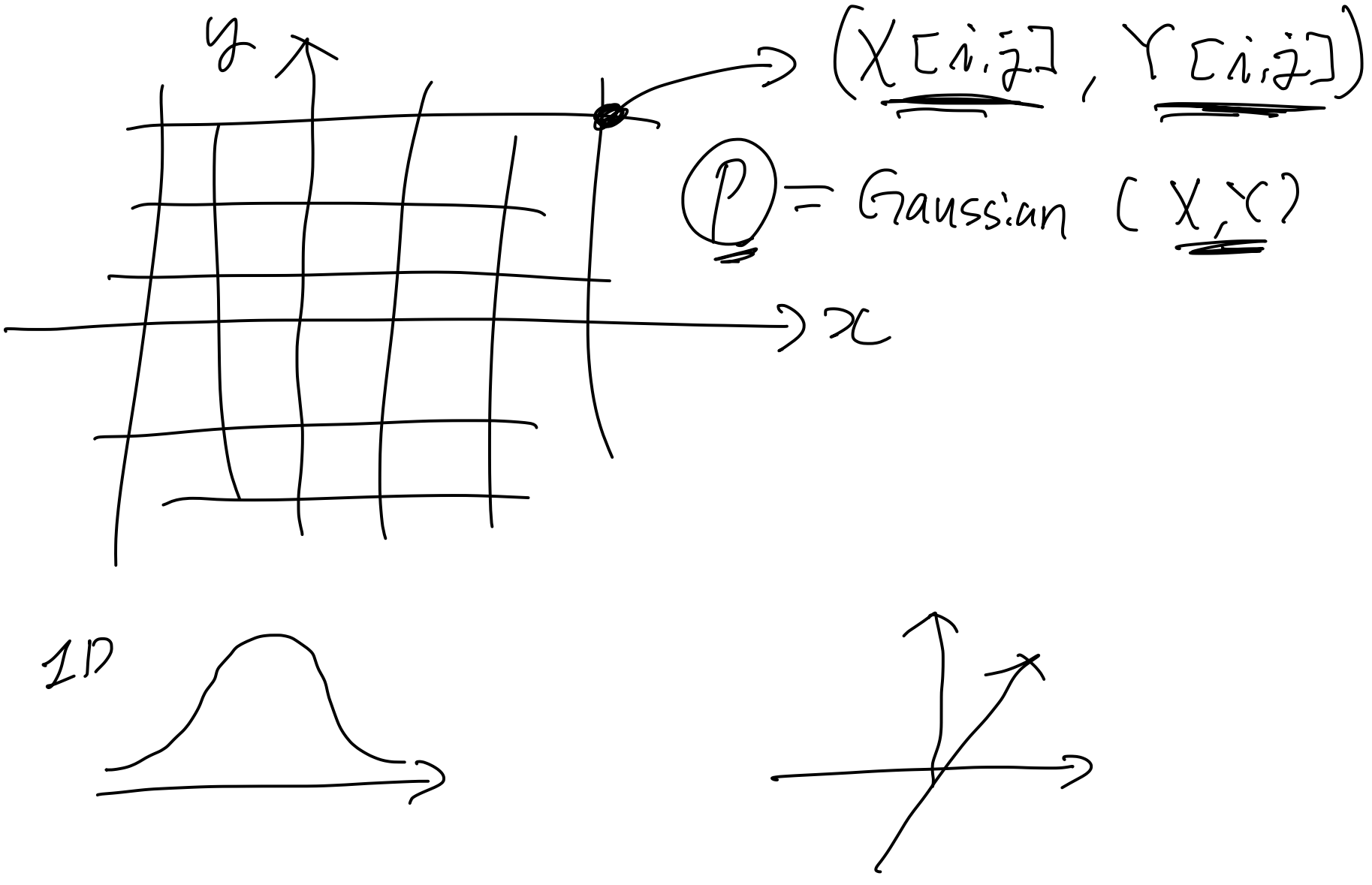
$X, Y = \text{meshgrid}(\underbrace{\text{linspace}(0,1,10)}_{\text{가로축을 쪼개는거.}}, \underbrace{\text{linspace}(0,1,10)}_{\text{세로축을 쪼개는거.}})$

$X =$

$$\begin{bmatrix} 0 & 0.1 & 0.2 & \cdots & 1 \\ 0 & 0.1 & 0.2 & & \\ 0 & 0.1 & 0.2 & & \\ 0 & 0.1 & \vdots & & \\ 0 & 0.1 & \vdots & & \\ \vdots & \vdots & & & \end{bmatrix}$$

$Y =$

$$\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0.1 & 0.1 & 0.1 & & 0.1 \\ \vdots & & & & \\ 1 & & & & \end{bmatrix}$$


y
x
(X[i,j], Y[i,j])
P = Gaussian (X, Y)

1D

$$\mu_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma(z_{nk})\, x_n$$

$$X = \begin{bmatrix} 1 & 2 & 3 & 3 & 4 & 4 & \cdots \\ 1 & 1 & 2 & 2 & 1 & 2 & \end{bmatrix}$$

$D \times N$

gamma $[k, i] = [0.4 \quad 0.7 \quad 0.7 \quad 0.6 \quad 0.4 \quad 0.3 \quad \cdots]$

$1 \times N$

$$\frac{1}{N_k} \left\{ 0.4 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 0.7 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0.7 \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \cdots \right\}$$