

리본가냥, 2
chain rule.

Pre-Class 13: Chain Rule

[SCS4049] Machine Learning and Data Science

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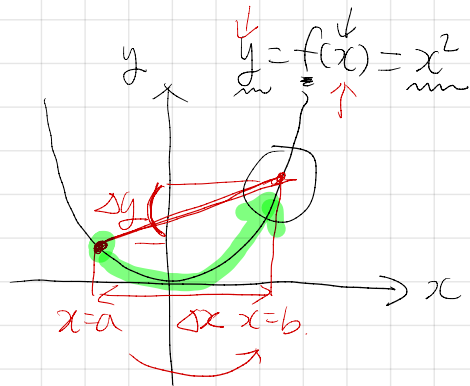
Chain rule

Chain rule: a formula for derivatives of the composition of two or more functions

$$\{f(g(x))\}' = f'(g(x)) \cdot g'(x) \quad (1)$$

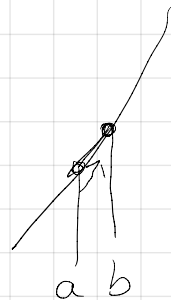
$$\frac{df}{dx} = \frac{df}{dg} \frac{dg}{dx} \quad (2)$$

미분 : 순간에서의 기울기



평균. 기울기. = $\frac{\text{종료값의 변화량}}{\text{시작값의 변화량}}$

$$= \frac{f(b) - f(a)}{b - a} = \frac{\Delta y}{\Delta x}$$



순간기울기
(b)

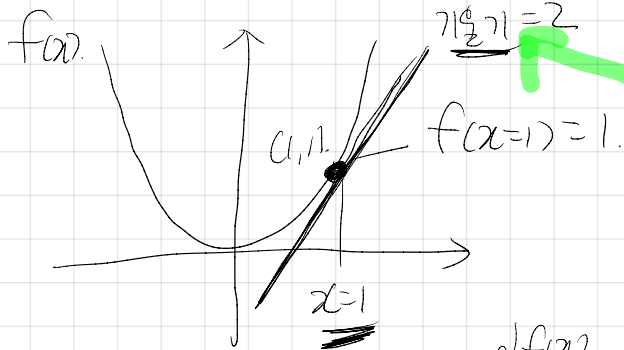
$$= \lim_{a \rightarrow b} \text{평균기울기}(a, b).$$

= 미분.

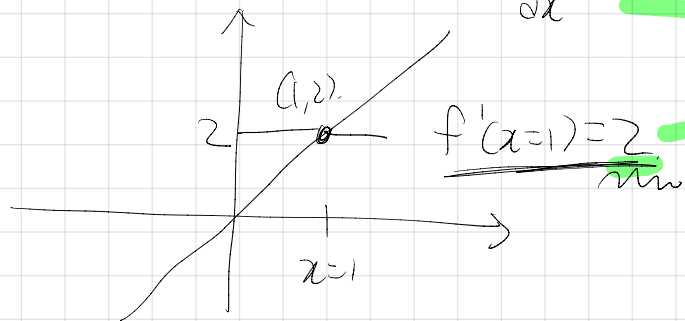
다항식 미분.

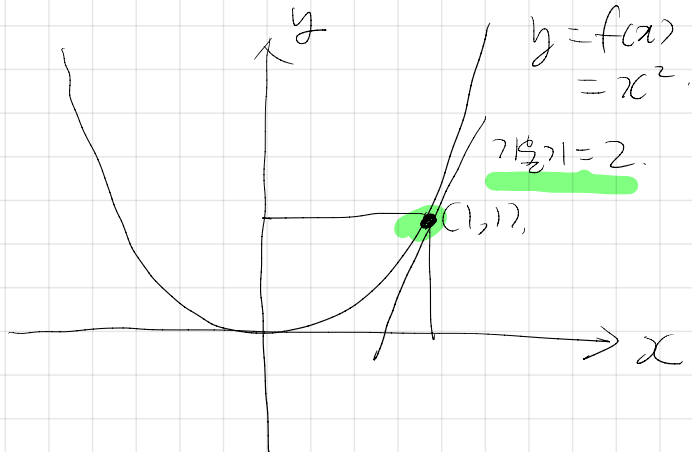
$$f(x) = x^2.$$

$$\frac{df(x)}{dx} = 2x.$$



$$\frac{df(x)}{dx} = f'(x) = 2x.$$





기울기 = $\frac{\text{출력의 변화량}}{\text{입력의 변화량}}$

입력이 +1만큼 변함 \Rightarrow 출력은 +2만큼 변함.

입력이 +0.1만큼 변함 \Rightarrow 출력은 +0.2만큼 변함.

입력이 Δ 만큼 변함 \Rightarrow 출력은 기울기 $\cdot \Delta$ 만큼 변함.
 미분값.

$$y = f(x) \quad \text{함수.}$$

$$f'(x), \quad \frac{df(x)}{dx}, \quad g', \quad \frac{dy}{dx}.$$

$$\boxed{\frac{dy}{dx}}$$

극소변리량.

$$\frac{dy}{dx} = \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta x \rightarrow 0}} \frac{\triangle y}{\Delta x}$$

$$\frac{dy}{dx} = \frac{\text{아직작은 } y \text{의 변리량}}{\text{아직작은 } x \text{의 변리량}} =$$

$$y = 2x^2 + x.$$

$$y = 2x + z = f(x, z).$$

$$\frac{dy}{dx} = 4x + 1$$

Chain Rule \Rightarrow 다단계 미분, 중간변수의 미분.

$$z = f(y), \quad y = g(x).$$

$$z = 2y, \quad y = 3x.$$

$$z = \underline{f(y)} = f(g(x)).$$

중간변수.

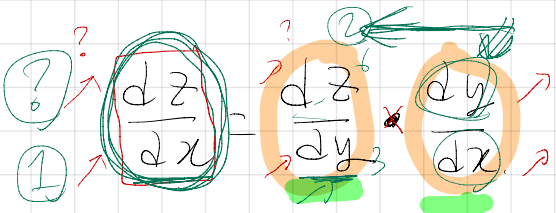
$$= \underline{2y} = 2(3x) = 6x$$

$$z = 2y \longrightarrow y = 3x \longrightarrow$$

$$\underline{z} = 2 \cdot (3x) = \underline{6x}$$

$$\begin{matrix} +6 \\ \textcircled{+6} \end{matrix} \cdot \frac{dz}{dy} = 2 \quad \times \quad \begin{matrix} +3 \\ \textcircled{+3} \end{matrix} \frac{dy}{dx} = 3$$

$$\frac{dz}{dx} = 6$$



$$= 2 \cdot 3$$

$$= \underline{6}$$

6

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$z \leftarrow \textcircled{y} \leftarrow x$$

$$z = 3y^2 + 2y$$

$$y = 6x^3$$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} = (6y + 2) \cdot (18x^2)$$

$$= (6 \cdot 6x^3 + 2) \cdot (18x^2)$$

$$= (36x^3 + 2) \cdot (18x^2)$$

$$z = 3y^2 + 2y = 3(6x^3)^2 + 2 \cdot (6x^3) = \underline{3 \cdot 36x^6 + 12x^3}$$

$$\frac{dz}{dx} = \underline{6 \cdot 3 \cdot 36x^5 + 36x^2}$$

$$z = f(y) \quad y = f(x) \quad x = f(u) \quad u = f(v) \dots$$

$$\frac{dz}{dv} = \frac{dz}{dy} \cdot \frac{dy}{dx} \cdot \frac{dx}{du} \cdot \frac{du}{dv}$$

(The above equation is annotated with red circles containing question marks and plus signs around the differentials and the chain rule structure.)

